**SAMSKRUTI COLLEGE OF ENGINEERING AND TECHNOLOGY**

**(Approved By AICTE & Affiliated to Jawaharlal Nehru Technological University Hyderabad)**

Kondapur (V), Ghatkesar (M), Ranga Reddy District, Hyderabad – 501301.

**DEPARTMENT OF H&S**

**II Year / I Semester**

**Name of the faculty:** Thirupathireddy **Designation:** Assistant professor

**Course Code:**MA301BS **Subject Title: Mathematics-4**

**Year & Semester:**  II Yr & I SEM  **Class:** B. Tech - CSE

**UNIT- I**

**FUNCTIONS OF A COMPLEX VARIABLE**

**Syllabus:**

Introduction,Countinuity,Differentiability,Analyticity,Properties,Cauchy-riemann equation in Cartesian co-ordinates.Harmonic and conjugate harmonic functions-Milne-Thompson method

**Objectives:**

**The objective of this course is introduce the fundamental ideas of the functions of complex variables developing a clear understanding of the fundamental concepts of complex analysis.**

**Lecture Plan:**

|  |  |  |
| --- | --- | --- |
| **S.No.** | **Topic** | **No. of Lectures** |
| 1 | Introduction | 2 |
| 2 | Countinuity | 1 |
| 3 | Analytic function | 2 |
| 4 | Cauchy-riemann equation in Cartesian coordinate | 4 |
| 5 | Cauchy-riemann equation in Polar coordinate | 2 |
| 6 | Harmonic and conjugate harmonic functions | 2 |
| 7 | Milne-Thompson method | 2 |

**Assignments:**

1. **State and prove Cauchy-Riemann equations in Cartesian co-ordinates**
2. **Find the analytic function Whose real part is u=e2x(xcos2y-ysin2y)**
3. **Prove that zn (n is a positive integer) is analytic and hence find its derivative**
4. **If f(z) is a regular function of z , prove that **

**Important Questions:**

1. **State and prove Cauchy-Riemann equations in Cartesian co-ordinates**
2. **State and prove Cauchy-Riemann equations inPolar co-ordinates**
3. **Show that the real and imaginary parts of the function w=logz**

**Satisfy the C-R equations when Z is not zero**

1. **Show that  is not analytic in the complex plane**
2. **Find whether f(z)=sinxsiny-icosxcosy is analytic or not**
3. **Find whether is not analytic or not**
4. **Show that the function  is not analytic at the origin , although C-R equations are satisfied at that point**



1. **Prove that the function f(z) defined by f(z)={**



**Is continuous and the C-R equations are satisfied at the origin,yet f’I(0) does not exist.**

1. **Show that an analytic function with constant absolute value (or modulus) is constant**
2. **Show that both the real and imaginary parts of an analytic function satisfies Laplace’s equation(0r Haromonic)**
3. **Show that (i) f(z)=ez (ii)  is nalytic everywhere in the complex plane and find fI(z)**

1. **Prove that zn (n is a positive integer) is analytic and hence find its derivative.**
2. **Prove that the function  is not analytic at any point.**
3. **Show that the function f(z)=sinxcoshy+icosxsinhy is continuous as well as analytic every where**
4. **Find k such that f(x)=x3+3kxy2 may be harmonic and find its conjugate.**
5. **Find the analytic function whose real part is (i)  (ii) **
6. **Find the analytical function whose real part is **
7. **Find the analytic function Whose real part is u=e2x(xcos2y-ysin2y)**
8. **find the conjugate harmonic of u= cos2xy .Hence find f(z) in terms of z**
9. **If f(z) =u+iv is an analytic function of z and if u-v =ex(cosy-siny) , find f(z) in terms of z**
10. **Find whether the function is Harmonic . If so, find the analytic function whose real part is u**
11. **Show that the function u=e-2xysin(x2-y2) is harmonic, find the conjugate function ‘v’ and express u+iv as an analytic function of z**

**UNIT II**

**COMPLEX INTEGRATION**

**Syllabus:**

**Line integral,Cauchy’s integral theorem,Cauchy’s integral formula and Generalized Cauchy’s integral formula,Power Series- Taylor’s series -Laurent series, Singular points,Isolated singular points,pole of Order m-Essential singularity,Residue,Cauchy Residue theorem(without proof)**

**Objectives:**

|  |  |  |
| --- | --- | --- |
| **S.No.** | **Topic** | **No. of Lectures** |
| 1 | **Line integral** | 2 |
| 2 | **Cauchy’s integral theorem** | 1 |
| 3 | **Cauchy’s integral formula** | 1 |
| 4 | **Generalized Cauchy’s integral formula** | 1 |
| 5 | **Power Series** | 2 |
| 6 | **Taylor’s series** | 1 |
| 7 | **Laurent series** | 1 |
| 8 | **Singular points** | 2 |
| 9 | **Isolated singular points** | 1 |
| 10 | **pole of Order m** | 2 |
| 11 | **Essential singularity** | 2 |
| 12 | **Residues** | 1 |
| 13 | **Cauchy Residue theorem** | 2 |

**The course is to introduce the fundamental ideas of the functions of complex variables and developing a clear idea of the fundamental concepts of complex integration**

**Assignments:**

1. **Evaluate**  **where c is the arc of the cycloid x=a(Ө+sinӨ) , y=a(1-cosӨ) between the points (0,0) and (aπ ,2a)**
2. **Evaluate**  **alog (1-i) to(2+i)**
3. **State and prve Taylor’s theorem**
4. **If c is boundery of the square with vertices at the points z=0 , z=1 , z=1+i and z=i Show that** .

**Important Questions:**

1. **Evaluate**  **where c consists of the line segments** from z=0 to z=i and the other form z=i to z=1+i
2. **Evaluate** along the paths (i) **y=x (ii) y=x2**
3. **Evaluate**  **where c is the arc of the cycloid x=a(Ө+sinӨ) , y=a(1-cosӨ) between the points (0,0) and (aπ ,2a)**
4. **Evaluate**  **alog (1-i) to(2+i)**
5. **Evaluate** where C is the boundary of the region by y=x2 and x=y2
6. **Evaluate**  along y=x2
7. **Evaluate**  **alog y=x2**
8. **Evaluate** **along the straight line from(2,0) to (2,2) and then from (2,2) to (0,2)**
9. **Evaluate** Where C is the contour consisting of the straight line from z=-i to z=i
10. **State and prove Cauchy integral theorem**
11. **Prove that**  , Where C is 
12. **Evaluate**  around (a)  **The circle  (b) The square with vertices 2±2i , -2±2i**
13. **If c is boundery of the square with vertices at the points z=0 , z=1 , z=1+i and z=i Show that** .
14. **LC C denote the boundary of the square whose sides lie along the lines x=±2 ,y=±2 where C in the positive sense .**
15. **Evaluate each of the following integrals.**

(a) **(b)** 

1. **Evaluate (i)**   **(ii)**  around c: .
2. **Evaluate**  where C is the circle 
3. **Evaluate** **where c is the circle (i)=1 (ii)** 
4. **Use Cauchy’s integral formula to evaluate**  Where C is the circle 
5. **Evaluate using Cauchy’s theorem  where C is .**
6. **Evaluate ,Where C is the circle **
7. **Evaluate  where c: Using Cauchy’s integral formula**
8. **Find f(2) and f(3) if f(a) = where C c is the circle  Using Cauchy’s integral formula**
9. **Evaluate  where C:  taken in anti-clock wise sense.**
10. **State and prve Taylor’s theorem**
11. **Prove that when **
12. **Withinwhat circle does the Maclaurin’s series for the function tanhz converge to the function.**
13. **Find Taylor’s series expansion for the function  with center at –i**
14. **Obtain the Taylor’s series to represent the function , in the region **
15. **Expand sinhz by Taylor’s series about z=πi.**
16. **State and prove Laurent’s series**

**Give two Laurent’s series expansions in powers of z for  and specify the regions in which these expansions are valid.**

**32 Expand  in the region (i)  (ii) **

**Find the Laurent’s series expansion of the function  in the region **

**33 Expand  about the point in the region  as Laurent’s series.**

1. **The only singularites of a single valued f(z) are poles of order 1 and 2 at z=-1 and z=-2 with residues at these poles 1 and 2 respectivly . If determine the function.**
2. **Represent the function  in Laurent series within **
3. **Find the residue of  at its pole**

**UNIT- III**

**EVALUATION OF INTEGRALS**

**Syllabus:** Types of real integrals

a)Improper real integrals  b) 

**Bilinear transformation-fixed point-cross ratio-properties-invariance of circles**

**Objectives: Here we consider the evaluation of certain types of real definite integrals .These integrals often arise in physical problems .The process of evaluating a definite integral by machining the path of integration about a suitable curve in the complex plane**

**Lecture Plan:**

|  |  |  |
| --- | --- | --- |
| **S.No.** | **Topic** | **No. of Lectures** |
| 1 | Improper real integrals | 1 |
| 2 |  | 3 |
| 3 | **Bilinear transformation-** | 2 |
| 4 | **fixed point** | 3 |
| 5 | **cross ratio** | 3 |

**Assignments:**

1.Show **that** 

2. Evaluate 

3.Evaluate using Residue theorem

4.Evaluate  using Residue theorem

**Important Questions:**

**1.Show that  using Residue theorem**

**2. Evaluate  using Residue theorem**

**3 . Show that **

**4. Show that **

**5 . Using contour integration , evaluate **

**6.Show that **

**7. Evaluate **

**8.Evaluate using Residue theorem**

**9. valuate  using Residue theorem**

**10. Evaluate  using Residue theorem**

**11.Evaluate by contour integration **

**12.Prove that **

**13.Evaluate **

**14.Evaluate **

**15. Evaluate** 

16. **By the method of contour integration,prove that **

**17.Find the points at which w=coshz is not conformal**

**18. Find the bion which maps the points z=1 ,i ,-1 onto the points w=i ,0 ,-i Hence ,find;**

1. **The image of ,**
2. **Concentric circles **
3. **The invariant points of the transformation.**

**19. The bilinear transformation which maps the points (-1,0,1) into the points(0,i,3i)**

**20. find the bilinear transformation which maps the points**  into the points 

21. **Find the fixed points of the transformation (i) **

**(ii) **

**(iii) **

1. **. Find the bion which maps the points  into the points respectively.To what curve the y-axis is transformed by this transformation**
2. **Show that the transformation  transforms the circle  in the z-plane into the imaginary axis in the w-plane**
3. **Show that the transformation  transforms the circle  into the real axis in the w-plane and the interior of the circle into upper half of the w-plane**

**UNIT- IV**

**FOURIER SERIES AND TRANSFORMS**

**Syllabus:**

**Introduction, Periodic functions , Fourier series of Periodic function,Dirchlet’s conditions,Even and odd functions,Change of interval,Half range sine and cosine series.Fourier integral theorem (without proof),Fourier sine and cosine integrals,Sine and Cosine Transforms,Properties,inverse transforms, Finite Fourier transforms**

**Objectives:**

**Fourier transforms are used in solving the PDE with boundary conditions**

**Lecture Plan:**

|  |  |  |
| --- | --- | --- |
| **S.No.** | **Topic** | **No. of Lectures** |
| **1** | **Introduction** | **1** |
| **2** | **Periodic functions** | **2** |
| **3** | **Fourier series of Periodic function** | **3** |
| **4** | **Dirchlet’s conditions** | **2** |
| **5** | **Even and odd functions** | **1** |
| **6** | **Change of interval** | **1** |
| **7** | **Half range sine series** | **1** |
| **8** | **Half range cosine series** | **1** |
| **9** | **Fourier integral theorem** | **1** |
| **10** | **Fourier sine integrals** | **2** |
| **11** | **Fourier cosine integrals** | **2** |
| **12** | **Sine Transforms** | **2** |
| **13** | **Cosine Transforms** | **1** |
| **14** | **Sine and Cosine Transforms properties** | **1** |
| **15** | **Inverse transeforms** | **1** |
| **16** | **Finite Fourier transforms** | **1** |

**Assignments:**

1. **Find the fourier series to represent f(x)=x2 in (0,2π)**
2. **Find a fourier series to represent the function f(x)=ex, for –π<x<π and hence derive a series for** 
3. **Find the fourier expansion of f(x)=xcosx; 0<x<2 π**
4. **Find the fourier series of period the function f(x)=x2-x in (-π, π).**

**Important Questions:**

**Assignments:**

1. **Find the fourier series to represent f(x)=x2  in (0,2π)**
2. **Express f(x)=x as afourier series in (-π, π).**
3. **Express f(x)=x2 as afourier series in (-π, π).**
4. **Express f(x)=x3 as afourier series in –π<x<π**
5. **Expand the fourier series to represent the function**
6. **If f(x)=coshx , expand f(x) as a fourier series in(-π, π).**
7. **Obtain fourier series for the function f(x)=**  in **(-π, π).**
8. **Obtain the half-range sine and cosine`for the function fourier** in the range
9. **Obtain fourier expansion for sinax in the interval –l<x<l**
10. **Find the fourier series for f(x)=2lx-x2  in (0,2l) Find a fourier series to represent the function f(x)=ex, for –π<x<π and hence derive a series for** 
11. **Find the fourier expansion of f(x)=xcosx; 0<x<2 π**

**Find the fourier series of period the function f(x)=x2-x in (-π, π)**

1. **Find the half-range sine series of f(x)=1 in [0,l]**
2. **Obtain the half-range cosine and sine series for f(x)=x in the interval**
3. **Find the fourier sine series for f(x)=2x-x2 , in 0<X<3 and f(x+3)=f(x)**
4. **Find the fourier transform of f(x** 
5. **Find the fourier transform of**  
6. **Find the sine and cosine transform of xe-ax**
7. **Find the finite fourier sine transform and cosine transform of f(x)=1**
8. **Find the Fourier sine transform of** **  **,0<x<π**
9. **If finite Fourier sine transform of f is then find f(x)**
10. **Find the inverse finite fourier sine transform f(x)=if where n is a positive integer and 0<x<8**

**UNIT- V**

**APLICATIONS OF PDE**

**Syllabus:**

**Classification of Second order partial differential equations,Method of separation of variables,Solution of one dimensional wave and heat equations**

**Lecture Plan:**

|  |  |  |
| --- | --- | --- |
| **S.No.** | **Topic** | **No. of Lectures** |
| 1 | **Classification of Second order partial differential equations** | 3 |
| 2 | **Method of separation of variables** | 1 |
| 3 | **Wave equation** | 2 |
| 4 | Heat equation | 2 |

**Assignments:**

1. **A tightly stretched string of length l has its ends fastends at x=0,x=l.The mid-point of the string is then taken to height h and then released from rest in that position.find the lateral displace ment of the string at time t from the instant of release**
2. **A string is stretched and fastened to two points at x=0 and x=l motion is started by displacing the string into the form y=k(lx-x2) from which it is released at time t=0.Find the displacement of any point on the sting at a distance of x from one end at time t**
3. **Slove by the method of separation of variables** 
4. **.Solve the method of separation of variables  where u(x,0)=6e-3x**
5. **Solve** 

**Importance Questions:**

1. **Solve the method of separation of variables 2xzx-8yzy**
2. **Using method of separation of variables, solve uxt=e-tcosx with u(x,0)=0 and u(0,t)=1**

**3.**Solve given that u=0 and when x=0,show that as t tends to  ,u tends to sinx. **A tightly stretched string of length l has its ends fastends at x=0,x=l.The mid-point of the string is then taken to height h and then released from rest in that position.find the lateral displace ment of the string at time t from the instant of release**

**4 . A string is stretched and fastened to two points at x=0**

**and x=l motion is started by displacing the string into the**

**form y=k(lx-x2) from which it is released at time t=0.Find**

**the displacement of any point on the sting at a distance of**

**x from one end at time t**

**5. Slove by the method of separation of variables**



**6.Solve the method of separation of variables**  **where**

**u(x,0)=6e-3x**

**7.Solve** 

**8.Find the temperature u(x,t) in a bar OA of length l which is**

**perfectly insulted laterally and whose ends ) and A are kept**

**at** ,**given that the intial temperature at any point P of the rod is given**

**as u(x,0)=f(x)**

**9.Solve the one dimensional heat flow equation** given that **u(0,t)=0 ,u(l,t), t>0 and** 

**10.An insulated rod OA of length l with insulated sides, has**

**intial temperature u(x,0)=f(x) for 0 ≤ x ≤ l the ends are**

**insulated at t=0.Find the subsequent temperature distribution.**

**11.An insulated rod of length L has its ends A and** B manintained at ,and 1000C respectively until steady state conditions prevail.If B is suddenly reduced to  and maintained at,find the temperature at a distance xfrom A at time t.